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DIFFUSION APPROXIMATION FOR FLUIDIZED-BED COAL COMBUSTION

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A one-dimensional model has been constructed for stationary coal combustion, which is based on equations for the oxidant balance in the continuous and discrete phases together with the energy conservation equation for the burning particles in the fluidized bed. The model has been identified from measurements and parameters have been determined such as the mean particle size, the activation energy, and the gas and solid transport coefficients.

A fluidized-bed furnace opens up good economic prospects because it can burn a wide range of fuels efficiently while meeting tightening specifications on pollutant discharge. In such a furnace, the coal particles burn in an inhomogeneous fluidized bed of noncombustible material (the large fraction from the ash residue). On a two-phase model, an upward gas flow is necessary to start the fluidization, which breaks through as bubbles (discrete phase). The model concepts are fruitful and enable one to analyze commercial catalytic fluidized-bed reactors in which relatively slow heterogeneous reactions occur [1]. With a fast reaction, such as the combustion of solid fuel in a fluized bed, the model requires refinement. It has been suggested [2] that the gas bubbles are partially filled with burning particles and that the oxidation occurs in both phases.

A jet model is used for the combustion of coal having a high volatile content and entering the fluidized bed through the gas-distributing grid. The volatiles rapidly released from the fuel form combustible-gas jets at the grid, while the oxygen from the continuous phase diffuses to the jet boundaries [1].

A two-continuum model may be used to describe solid-fuel combustion in a fluidized bed composed of relatively coarse material, in which it is assumed that the entire gas flow is in contact with the solid and that no gas bubbles break through [3].

Here we present a three-continuum model. The bed consists of finely divided material (solid) suspended by the gas together with the gas, while in turn, the gas flow is divided into two continuous ones in accordance with the two-phase hydrodynamic fluidization theory: one is the gas passing through the channels between the grains (continuous phase) and the second is the gas in bubbles (discrete phase). A system of one-dimensional stationary nonlinear balance equations is formulated. The task if first simplified somewhat on the basis that the granular material mixes rapidly and therefore the burning-particle concentration and temperature remain virtually constant and are independent of the depth. One can therefore assume that the combustion rate does not vary with depth.

We write the conservation equation for the oxidant in the continuous and discrete phases:

$$k\varepsilon Y'' - \frac{1}{N-1} Y' - B\bar{\xi}\varphi YC + P\Pi (Y_d - Y) = 0, \tag{1}$$

$$Y_d' + P\Pi (Y_d - Y) = 0, \tag{2}$$

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and the energy conservation equation for the burning particles mixing in the bed:

$$\theta'' + q\varphi m\xi - \Phi B \frac{\varphi}{k} \theta = 0. \qquad (3)$$

It is assumed that the fuel burns completely at the particle surface with the formation of carbon dioxide at a rate governed by a standard formula [4, 5] on the basis of the oxidant diffusing to the surfaces;

$$\frac{1}{\xi} = \frac{K_* d_i}{\mathrm{sh}\, D} + \exp \frac{E}{RT_b} \left[\frac{1}{1 + \theta(1 - T_0/T_b)} - \frac{T_b}{T_*} \right]. \qquad (4)$$

We introduce dimensionless combinations:

$$B = \frac{6K_* H}{d\varphi (U - U_0)}, \quad m = \frac{\rho_*}{\rho_p(1-\varepsilon)}, \quad \theta = \frac{T_p - T_b}{T_b - T_0}, \quad \mathrm{Pe} = \frac{W}{U - U_0},$$

$$\Pi = \frac{\beta_d H}{U - U_0}, \quad q = \frac{Q_w^\ell}{c_p(T_b - T_0)}, \qquad (5)$$

$$\Phi = \frac{\mathrm{Nu}\,\lambda/d_i + 4\varepsilon_r C_0 T_b^3}{K_* \rho_p c_p (1-\varepsilon)}.$$

The parameters appearing here are governed by empirical relationships:

$$\mathrm{Nu} = 0.41\,\mathrm{Ar}^{0.3} \left(\frac{d_i}{d}\right)^{0.2} \left(\frac{\rho_p}{\rho_i}\right)^{0.07}, \qquad (6)$$

$$\varepsilon = \varepsilon_0 (1 + 0.5\,\mathrm{Fr}^{0.27}), \qquad (7)$$

$$Sh = 2.78\,\mathrm{Re}^{1/3} Sc^{1/3} \left(\frac{\rho_p}{\rho_i}\right)^{0.15} \left(\frac{d_i}{d}\right)^{0.13}, \qquad (8)$$

$$\beta_d = 520 \left(\frac{U^2}{gH}\right)^{0.53} \left(\frac{\rho}{\rho_i}\right)^{0.53} \mathrm{Ar}^{-0.15}, \qquad (9)$$

$$D_T = k(U - U_0) H. \qquad (10)$$

The conservation equations incorporate the oxidant diffusing in the continuous phase and the organized oxidant transport in the two phases, as well as the oxidant absorption in the continuous phase and the oxygen exchange between the two phases. The energy-conservation equation for the burning particles reflects the trends in heat production at the surfaces and the transfer to the incombustible particles by conduction, convection, and radiation, as well as the heat diffusing through the bed with the moving burning particles. There is also a term describing the heat diffusion with the burning particles and the heat generation and transport to the incombustible particles by those three mechanisms.

The boundary conditions are:

$$x = 0, \quad Y' = \frac{Y - Y_0}{k(N-1)}, \quad Y_d = Y_0 = 0{,}21, \quad \theta' = 0, \qquad (5')$$

$$x = 1, \quad Y' = 0, \quad \theta' = -\frac{\mathrm{Pe}}{k}. \qquad (6')$$

At the lower boundary of the bed, the organized flux in the continuous phase is equal to the diffusion one, while the oxidant concentration in the discrete phase is equal to the oxygen content on the incoming air. At the upper boundary, there is no oxidant transport in the continuous phase and there is a negative heat source produced by the entering cold fuel.

TABLE 1. Model Identitifaction from Measurements from Fluidized-Bed Coal Combustion

№	Model	Parameters					
		E, kJ/mole	φ	k	P	m	F_t
1	Two-continuum [3]	109,5	2,2	0,108	—	—	6,455
2	Three-continuum	97,5	3,2	0,132	4,99	—	6.01
3	Generalized	109,2	1,75	0,107	5,09	2,1	5,885

The initial system (1)-(3) was reduced to five first-order linear equations containing constant coefficients, which were solved by standard methods [11], the solution being

$$Y = \sum_{l=1}^{5} \bar{C}_l \frac{K_l^2 - b_{5,4}}{b_{5,1}} \exp K_l x, \tag{7'}$$

$$Z = \frac{dY}{dx} = \sum_{l=1}^{5} \bar{C}_l \frac{K_l^2 - b_{5,4}}{b_{5,1}} K_l \exp K_l x, \tag{8'}$$

$$Y_d = \sum_{l=1}^{5} \bar{C}_l \frac{b_{3,1}}{b_{5,1}} \frac{K_l^2 - b_{5,4}}{K_l - b_{3,3}} \exp K_l x, \tag{9'}$$

$$\theta = \sum_{l=1}^{5} \bar{C}_l \exp K_l x, \tag{10'}$$

$$\psi = \frac{d\theta}{dx} = \sum_{l=1}^{5} K_l \bar{C}_l \exp K_l x, \tag{11}$$

in which K_ℓ are the roots of the characteristic equation, $b_{3,1} = P\Pi$, $b_{3,3} = -P\Pi$, $b_{5,1} = -qm\bar{\xi}B\varphi/k$, $b_{5,4} = \Phi\varphi B/k$, $b_{2,1} = (BC_{01}\bar{\xi}\varphi + P\Pi)/(k\varepsilon)$. The constants of integration $\bar{C}_\ell$ are derived from boundary conditions (5') and (6').

The solution is dependent on the mean combustion rate, which from (4) is in turn dependent on the particle temperature (superheating). We integrate (4) with (10') to get

$$\left\langle \frac{1}{\xi} \right\rangle = \int_0^1 \frac{dx}{\xi} = F_1(\bar{\xi}) \tag{12}$$

of $\frac{1}{\bar{\xi}} - F_1(\bar{\xi}) = 0$.

This was solved by the dichotomy method to derive the mean burning rate, so (1)-(3) was solved together with (4), which enables one to relate the mean burning rate to the superheating.

System (1)-(3) contains unknown parameters: the effective activation energy for the combustion E, the mean relative dimensionless diffusion coefficient k, the mean relative burning particle size $1/\varphi$, and the relative phase gas transfer coefficient P. These were determined by identifying the model from measurements, which have been described in detail previously [3, 12-14]. The target-function minimum was derived by means of a deformable polyhedron; that function was the square of the difference between the observed and calculated excess-air coefficients at the exit from the bed:

$$F_t = \sum_{n=1}^{m_0} (\alpha_{N\,e}^n - \alpha_{N\,t}^n)^2, \tag{13}$$

in which

$$\alpha_{N_t} = \left[1 - \left(1 - \frac{1}{N}\right)\frac{Y_d}{0.21} - \frac{Y}{0.21}\right]^{-1}.$$

The oxygen oxidizes the fuel quite rapidly. In a thin fluidized bed, this occurs largely near the grid, where gas bubbles are formed (discrete phase), and where the gas emerging from the grid is distribued between the two phases. In the discrete one, the gas flow rises from zero at x = 0 to N − 1 at x = 1, while that in the continuous one falls from N at x = 0 to one at x = 1. We assume that the proportion of the gas moving in the continuous phase is

$$\gamma = (1 - N)x^m + N$$

and the mean value is

$$\bar{\gamma} = \int_0^1 \gamma dx = \frac{1 + mN}{m + 1}.$$

This gives us the general model in which the proportion of gas entering the continuous phase is not fixed but instead in an unknown parameter. This merely modifies the (1) and (2) oxidant balance equations because there are changes in the flows of ordered gas in the two phases. Instead of (1) we have

$$k\varepsilon Y'' - \frac{\bar{\gamma}}{N - 1} Y' - \xi\varphi BCY + P\Pi(Y_d - Y) = 0. \quad (14)$$

Similarly, instead of (2) we have

$$\frac{N - \bar{\gamma}}{N - 1} Y_d' + P\Pi(Y_d - Y) = 0. \quad (15)$$

Table 1 gives these parameters calculated from a single measurement set derived for long-flame coal from the Donetsk deposit in a system at the Institute of Heat and Mass Transfer, Belorussian Academy of Sciences (water content 3.7%, ash 23.6%, and heat of combustion 23.2 MJ/kg) [12, 13].

The target function is least for model 3, so it best fits these this beds. The calculated values are quite likely. For example, the effective activation energy coincides within 15-17% with tabulated values [4]. The mean relative size of the burning particles is 0.57, which hardly differs from the arithmetic mean 0.5. The relative effective diffusion coefficient is close to that determined in independent experiments on solid mixing in a fluidized bed [10].

Interesting results were obtained on gas exchange. These beds up to 0.3 m deep gave values for the transfer coefficient appreciably higher than one, while those obtained by other researchers [9] up to 400°C were approximately one.

This general model has been identified by experiment and can be recommended for determining temperature and oxygen-concentration patterns for the continuous and discrete phases.

NOTATION

C, fuel concentration in fluidized bed; C_0, black-body emissivity; c_p, specific heat of fuel; D, gas diffusion coefficient; d and d_i, particle diameters for fuel and inert material; E, activation energy; H, bed depth; K_*, combustion rate constant; N, fluidization number; k, effective dimensionless mixing coefficient; Q_w^l, lower heat of combustion for working mass of fuel; R, universal gas constant; T_p, T_0, and T_b, temperatures of hot particle, incoming cold coal, and bed; T_m, characteristic temperature for coal combustion; U and U_0, infiltration speed and onset of fluidization speed; Y, oxygen concentration in continuous phase; Y_d, the same for the discrete phase; W, coal supply rate; α heat-transfer coefficient from burning particle to bed; α_N, excess air coefficient; β, mass transfer coefficient; γ, proportion of gas flow in continuous phase; ε, bed porosity; ε_r, reduced degree of blackness; λ, gas thermal conductivity; ξ, relative burning rate; ρ_p, ρ, ρ_*, densities of coal particles, inert material, and gaseous carbon; $1/\varphi$, mean relative burning particle diameter.

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CALCULATING SOLID-FUEL CONSUMPTION IN A CIRCULATION SYSTEM

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A size-distribution equation is used in deriving the burning rate for a polydisperse solid fuel subject to repeated external circulation through a cyclone. The effects from fuel fractional composition and cyclone parameters are examined.

Recently, increasing use has been made of systems for burning solid fuel involving repeated circulation through a cyclone (Fig. 1), as with a circulating fluidized bed [1] or in an air jet furnace [2]. Fresh fuel is supplied to the combustion chamber from a dust-handling device at a rate G_{a0}, while partially burned (secondary) fuel is fed from the cyclone at a rate G_{ac}, i.e., $G_a = G_{a0} + G_{ac}$. The fuel entering the cyclone from the combustion chamber G_b passes through it and returns to the combustion chamber as $G_{ac} = G_b - G_c$, with the exception of the part G_c that is not trapped is lost from the system. G_c/G_b is dependent on the cyclone's performance and governs the mechanical incompleteness in the combustion.

It is difficult to perform calculations on such burning because the fuel fractional composition (size distribution) at the inlet to the combustion chamber is not known in advance and is depend on the composition of the cyclone material, i.e., on the size distribution for the particles entering the cyclone and the characteristics of the latter. The size distribution at the inlet is thus dependent on that at the outlet, and in that sense, the combustion is a self-consistent system; even if the input from the dust preparation device was monodisperse, the repeated circulation makes it polydisperse.

1. We consider coke particles burning in an air flow, with the particle density ρ_2 unaltered during the combustion. To consider a polydisperse system, we consider the kinetic equation for the size distribution used in [3, 4 for combustion in an ordinary chamber with-

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